Regularization of optimal control problems on stratified domains using additional controls

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In this presentation, we investigate a Mayer optimal control problem governed by a dynamics defined regionally. We consider that the state space is stratified into a family of disjoint regions with nonsmooth interfaces, and that in each region, the dynamics is given by a smooth expression as follows :

$$\dot{x} = f_j(x,u), \quad \text{if } \varphi_j(x) < 0.$$

First, it is shown that this problem is equivalent to a new optimal control problem, with additional controls v_j taking values in $[0,1]$ and a (smooth) dynamics as a convex combination of the smooth dynamics $\sum_{j=1}^{N} v_j f_j(x,u_j)$, along with the following mixed control-state constraint :

$$(1-2v_j)\varphi_j(x) = |\varphi_j(x)|.$$

Next, we introduce a family of auxiliary optimal control problems. In these problems, we first regularize the nonsmooth interfaces. In addition, we consider the convex combination of smooth dynamics (only) within a boundary layer. Furthermore, we add a penalization term to the cost function to account for the mixed control-state constraint. Our main result is that solutions to these (smooth) problems converge (up to a subsequence) to a solution of the original one. It is obtained thanks to a new hypothesis related to solutions to the auxiliary problems, which is weaker than the transverse crossing condition of the literature. This technique is implemented numerically on two examples involving non-transverse crossings of interfaces, showing its efficiency.

References

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