Space-Time Finite Volume Method with Anisotropic Mesh Adaptation for Numerical Fluid Mechanics

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In numerical simulations of fluid mechanics, achieving higher accuracy can be approached in several ways : refining the physical modeling of flows, increasing the resolution of discretization, or improving the precision of numerical methods. To balance computational cost and accuracy, anisotropic metric-based mesh adaptation has demonstrated its effectiveness for steady problems. However, extending this approach to unsteady flows remains a significant challenge due to the complexities introduced by time dependence. One key issue is the lagging of the mesh relative to the evolving solution. When a mesh is optimized for a solution at specified time, it quickly becomes suboptimal as the solution advances. Several methods in the literature [2] address this issue, but they require introducing complex interpolation techniques to transfer the solution between meshes, as well as intricate metric definitions to ensure accuracy. In this work, we will introduce a novel numerical method designed for unstructured meshes, ensuring conservation in both space and time, while mitigating these challenges in an efficient way.

The proposed method is based on a space-time finite volume formulation to solve the Euler equations on unstructured meshes. The solution domain is discretized in both space and time, forming space-time control volumes where the governing equations are integrated. Conservation is ensured by enforcing flux continuity across internal space-time faces. However, applying this method results in a nonlinear optimization problem due to the strong coupling between space and time. This problem is addressed using various solvers [1], while metric-based mesh adaptation is employed to maintain accuracy and stability. The method is validated for both first-order and second-order finite volume schemes. However, while the second-order extension is functional, improvements are still needed, particularly in the treatment of limiters, to fully enhance its accuracy and stability.

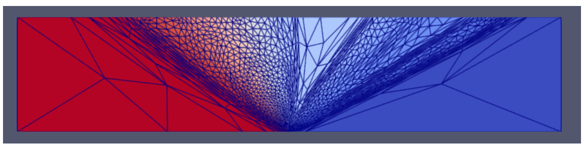

FIGURE 1 – Density : first-order scheme, Sod shock tube.

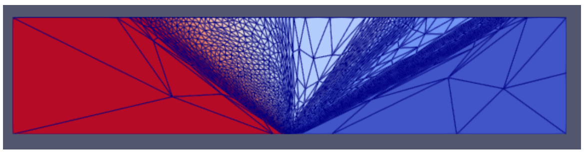

FIGURE 2 – Density : second-order scheme, Sod shock tube.

[1] P. Brune, M. G. Knepley, B. Smith, X. Tu. *Composing scalable nonlinear algebraic solvers.* SIAM Review, **30(4)**, 2015.

[2] F.Alauzet, A.Loseille, G.Olivier. *Time-accurate multiscale anisotropic mesh adaptation for unsteady flows in cfd.* Journal of Computational Physics, **36(3)**, 2018.

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