Positive finite volume scheme for a degenerate parabolic system describing chemotaxis

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In this presentation, we will talk about the discretization and the numerical analysis of degenerate parabolic system modeling chemotaxis [5]. This is done thanks the Discrete Duality Finite Volume (DDFV) scheme [1, 3]. The method is not positive in general [4]. To deal with this issue, we introduce nonlinear corrections to guarantee the positivity of the density and the concentration. The advantages of our approach are multiple. For instance, the angle condition is not imposed on the mesh, the anisotropy diffusion can be included, and the scheme's stencil remains unchanged.

For this purpose, the key terms of convection and diffusion are carefully approximated. The transport contribution is discretized using a combination of centered and upstream schemes, while the oscillatory diffusive fluxes are handled through an upwind scheme. By deriving a priori estimates, we demonstrate the convergence of the approximate solutions using compactness arguments that are designed to degenerate parabolic problems [2]. Typical numerical experiments will be shown in two dimensions to evaluate the efficiency and robustness of the proposed solver across various test cases, including anisotropic tensors.

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