# Asymptotic behavior of the solutions to the Gurtin - MacCamy's population model

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In this talk, we consider a family of nonlinear convolution equations which arise naturally in the study of the Gurtin - MacCamy's population model. Specifically, we investigate how much of the dynamics about the infinite dimensional system can be inferred from the one-dimensional dynamical system given by the nonlinearity. Under this framework, we present various conditions assuring that all solutions to this equation have asymptotic constant behavior [2]. These results provides sharp and easily verifiable criteria that guarantee the global attractivity of the unique nontrivial steady state of the Gurtin - MacCamy's population model, complementing previous works [3].
Furthermore, we explore the existence of periodic regimes in this model. As a first sight, and motivated by the earlier work [1], we consider a weakly delayed and non-symmetric case. Our analysis focuses on the properties of the first-return map, called the Poincaré map, defined on an suitable forward invariant set constructed by means of a barrier of subsolutions to the original equation. We show that this forward invariant subset possesses the fixed-point property by identifying it with a convex, closed subset of certain Banach space, thus allowing the application of classical fixed point theory.

[1] S.-N. Chow, O. Diekmann, J. Mallet-Paret. *Stability, multiplicity and global continuation of symmetric periodic solutions of a nonlinear Volterra integral equation.* Japan J. Appl. Math., **2(2)**, 433–469, 1985. doi :10.1007/BF03167085.

[2] F. Herrera, S. Trofimchuk. *Dynamics of one-dimensional maps and Gurtin-MacCamy's population model. Part I. Asymptotically constant solutions.* Ukrainian Math. J., **75(12)**, 1850–1868, 2024. Reprint of Ukraïn. Mat. Zh. **75** (2023), no. 12, 1635–1651.

[3] Z. Ma, P. Magal. *Global asymptotic stability for Gurtin-MacCamy's population dynamics model.* Proc. Amer. Math. Soc., **152(2)**, 765–780, 2024. doi :10.1090/proc/15629.

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