Sample and Map from a Single Convex Potential: Generation using Conjugate Moment Measures

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A common approach to generative modeling is to split model-fitting into two blocks : define first *how* to sample noise (e.g. Gaussian) and choose next *what* to do with it (e.g. using a single map or flows). We explore in this work an alternative route that *ties* sampling and mapping. We find inspiration in *moment measures* [1], a result that states that for any probability measure ρ, there exists a unique convex potential u such that $\rho = \nabla u \sharp e^{-u}$. While this does seem to tie effectively sampling (from log-concave distribution e^{-u}) and action (pushing particles through ∇u), we observe on simple examples (e.g., Gaussians or 1D distributions) that this choice is ill-suited for practical tasks. We study an alternative factorization, where ρ is factorized as $\nabla w^* \sharp e^{-w}$, where w^* is the convex conjugate of w. We call this approach *conjugate* moment measures, and show far more intuitive results on these examples. Because ∇w^* is the Monge map [2] between the log-concave distribution e^{-w} and ρ, we rely on optimal transport solvers to propose an algorithm to recover w from samples of ρ, and parameterize w as an input-convex neural network.

[1] D. Cordero-Erausquin, B. Klartag. *Moment measures.* Journal of Functional Analysis, **268(12)**, 3834–3866, 2015.

[2] G. Monge. *Mémoire sur la théorie des déblais et des remblais.* Histoire de l'Académie Royale des Sciences, pp. 666–704, 1781.

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