General selection principle for the Fleming-Viot process

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Given a diffusion X_t on the positive real line which hits 0 at time τ_0, a quasi-stationary distribution is a probability measure π such that $\mathcal{L}_\pi(X_t|\tau > t) = \pi$ for all $t > 0$. There may be zero, one or infinitely many QSDs. However, if there is at least one, then there exists a distinguished QSD π_* corresponding to the limit

$$\pi_* = \lim_{t\to\infty} \mathcal{L}_x(X_t|\tau_0 > t)$$

for all deterministic initial conditions $x > 0$.

The associated Fleming-Viot process consists of N particles evolving independently according to the dynamics of the given diffusion, with the additional rule that whenever a particle hits 0, it jumps to the location of another particle chosen independently and uniformly at random. These were introduced by Burdzy, Holyst and March in 2000 as a particle representation for quasi-stationary distributions.

It has been widely believed for around 20 years that the following selection principle holds. If the driving absorbed diffusion has a Yaglom limit, then the stationary empirical measure of the N-particle system converges as $N \to \infty$ to this Yaglom limit. We prove this conjecture.

This builds on earlier work of myself proving this conjecture for the particular case of constant diffusivity and constant negative drift, and of Julien Berestycki and I proving a similar longstanding conjecture for the related N-BBM. Whilst most of the techniques introduced in the proof of the former were very general, it relied on the precise description of the domain of attraction of the Yaglom limit available in that setting, which is not at all available more generally. The corresponding result is not at all available in the N-BBM context, but we were instead able to make use of the "stretching lemma" for Fisher-KPP equations first introduced by Kolmogorov, Petrovskii and Piskunov in 1937.

We will introduce an analogue of the stretching lemma for one-dimensional diffusions. This is the key additional ingredient allowing us to generalise the earlier selection principle to general diffusions on the positive real line.

This is work in progress.