Numerical simulations of a quasilinear Gross–Pitaevskii equation with vanishing and nonvanishing conditions at inifinity

Sebastián TAPIA, Centre Inria de l'Université de Lille, Laboratoire Paul Painlevé - Lille

We focus on the quasilinear Schrödinger equation

$$i\partial_t \Psi = \partial_{xx}\Psi + \mathfrak{s}\left(\Psi|\Psi|^2 + \kappa\Psi\partial_{xx}|\Psi|^2\right), \tag{QGP}$$

where $\Psi : \mathbb{R} \times \mathbb{R} \to \mathbb{C}$ denotes the complex-valued wave function, $\kappa \in \mathbb{R}$ with zero and nonzero conditions at infinity. Here κ is a parameter modulating the strength of the quasilinear contribution, and $\mathfrak{s} \in \mathbb{R}$ distinguishes between the focusing ($\mathfrak{s} > 0$) and defocusing ($\mathfrak{s} < 0$) regimes. This quasilinear model corresponds to a weakly nonlocal approximation of the nonlocal Gross–Pitaevskii equation, and can also be derived by considering the effects of surface tension in superfluids. In the focusing case, the existence and stability of bright solitons was established in [1]. On the other hand, in the defocusing case with nonvanishing conditions at infinity, a complete classification of finite energy traveling waves has recently been done in [2], leading to the existence of dark and antidark solitons, even for supersonic speeds. However, the well-posedness of this quasilinear Schrödinger equation is an open problem in the energy space.
Our goal is to provide numerical methods which preserves the energy and mass, to compute accurate approximations of the evolution of (QGP), with zero and nonzero conditions at infinity. We are particularly interested in: Stability of bright solitons under collisions varying κ, short-term blow-up in the defocusing case with zero background, long-term stability of dark solitons and short-term stability of antidark solitons. These simulations provide several conjectures about the dynamics of (QGP) in these different contexts.

[1] M. Colin, L. Jeanjean, M. Squassina. *Stability and instability results for standing waves of quasi-linear Schrödinger equations.* Nonlinearity, **23(6)**, 1353–1385, 2010. doi:10.1088/0951-7715/23/6/006.

[2] A. de Laire, E. L. Quiniou. *Exotic traveling waves for a quasilinear Schrödinger equation with nonzero background.* Preprint arXiv:2311.08918.

Contact : `sebastian.tapia@univ-lille.fr`