Boundary null-controllability of a system of coupled parabolic equations on the disk

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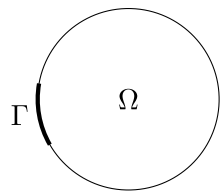
Given an ODE or a PDE, we may ask whether we can achieve a prescribed behavior for the solution by acting on the system through a control (e.g., a source term, a boundary term). This is the goal of controllability theory in a nutshell. For this talk I will be focusing on the following system.

$$\begin{cases} \partial_t y_1 - \Delta y_1 = 0 \text{ in } [0,T]\times\Omega, \\ \partial_t y_2 - \Delta y_2 + y_1 = 0 \text{ in } [0,T]\times\Omega, \\ y_1 = \mathbf{1}_\Gamma v \text{ in } [0,T]\times\partial\Omega, \\ y_2 = 0 \text{ in } [0,T]\times\partial\Omega, \\ y(0,\cdot) = y_0. \end{cases}$$



The questions we are interested in are whether this system is null-controllable, and if so, how much it costs. More precisely, given an initial data y_0 and a time T, we ask if there exists a control v such that the solution of the system satisfies $y(T) = 0$, and we aim to obtain a bound on the norm of v.
The main issue of this system is that we seek to control two components using a single control. This issue has been addressed in the more classical setting, when the control acts through a source term in a small region $\omega \subset \Omega$, and null-controllability has been proven for any domain Ω in any dimension. However, in our case, where the control acts through the boundary, the problem becomes more intricate. Techniques have been developped to handle the one-dimensional case (when Ω is an interval), and these can be used to prove null-controllability for certain specific multi-dimensional problems, when the domain is cylindrical. In this talk I will explain how to prove null-controllability when the domain is a disk (and more generally a ball in any dimension).

I will start my talk with an introduction about controllability theory, then I will explain the issues of this problem, and finally give some elements of proof. In a few words, the proof consists in proving null-controllability when $\Gamma = \partial\Omega$, with a good bound on the control using the moment method, and then applying a Lebeau-Robbiano type strategy to prove null-controllability in the general case.

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