Control of the half-heat equation.

Andreas HARTMANN, IMB - Bordeaux **Armand KOENIG**, IMB - Bordeaux

We know that the strong dissipation of the heat equation implies its null-controllability, thanks to the so-called *Lebeau-Robbiano's method*. In fact, if we consider the fractional heat equation $(\partial_t+(-\Delta)^\alpha)f = \mathbf{1}_\omega u$, this Lebeau-Robbiano's method works as long as $\alpha > 1/2$. On the other hand, null-controllability does not hold when $\alpha < 1/2$ or when $\alpha = 1/2$ in dimension 1.
In the case $\alpha = 1/2$ and dimension 1, the proof uses a natural connection between solutions of the half-heat equation and harmonic functions on the unit disk. In a joint work with Andreas Hartmann, we precise this result and study the space of null-controllable initial states. Leveraging tools from complex and harmonic analysis (Hardy and Bergman spaces, separation of singularities, Carleson measures, etc.), we prove

— the space of null-controllable initial states does not depend on time ;
— it is dense with dense complement in every $W^{s,2}$;
— it is a subset of the projection on positive frequencies of functions in $L^2(\omega)$.

Contact : armand.koenig@math.u-bordeaux.fr