A POD and Greedy Reduced Order Method for the stationary Neutron Diffusion Equation - Application to Nuclear Core computation in APOLLO3®

Amélie BRASSART, Université Paris-Saclay, CEA, Service d'Etudes des Réacteurs et de Mathématiques Appliquées - 91191, Gif-sur-Yvette, France.

Geneviève DUSSON, Laboratoire de Mathématiques de Besancon, UMR CNRS 6623, Université de Franche-Comté - 16 route de Gray, 25030, Besançon, France.

Virginie EHRLACHER, CERMICS, Ecole des Ponts ParisTech - 6-8 avenue Blaise Pascal, Cité Descartes, 77455, Cedex 2, France.

Tony LELIÈVRE, CERMICS, Ecole des Ponts ParisTech - 6-8 avenue Blaise Pascal, Cité Descartes, 77455, Cedex 2, France.

François MADIOT, Université Paris-Saclay, CEA, Service d'Etudes des Réacteurs et de Mathématiques Appliquées - 91191, Gif-sur-Yvette, France.

The deterministic resolution of the stationary neutron diffusion equation for nuclear reactor is computationally expensive, especially in a so-called many query context where this problem has to be solved for many different settings. This is for example the case to optimize the loading pattern, or to perform sensitivity calculations with respect to some physical parameters. Reduced Order Methods (ROM) can considerably reduce the computational cost of this parameter-dependent eigenvalue problem without much loss of accuracy. The time savings are significant in some neutronic applications.
In this work, a ROM is implemented in the nuclear reactor physics code APOLLO3® and tested on a benchmark called the MiniCore test case. Two methods of construction of the reduced basis, in the offline stage, are compared. For both, the construction uses a set of High Fidelity (HF) direct and adjoint fluxes snapshots. The first is a Proper Orthogonal Decomposition (POD) approach already tested in [2], which is computationally stable and provides the best (in some L^2 norm) reduced basis of size $N \ll \tilde{N}$ from a set of snapshots of size $\tilde{N}$. POD becomes prohibitively expensive when the number $\tilde{N}$ of snapshots is too large. The second method is a Greedy approach. The reduced basis is expanded by choosing iteratively the HF direct and adjoint fluxes from the training set, which are the less well approximated using the current reduced basis. The error estimate uses a (partially heuristic) *a posteriori* error estimator, studied and implemented in [1, Chapter 2], [3] : it allows for approximating the error without computing the HF fluxes in the training space, hence the computational gain compared to the POD approach. We will present and discuss the results obtained with the Greedy method.

[1] Y. Conjungo Taumhas. *Criticality calculations in neutronics : model order reduction and a posteriori error estimators.* Ph.D. thesis, École des Ponts ParisTech, 2023.

[2] Y. Conjungo Taumhas, G. Dusson, V. Ehrlacher, T. Lelièvre, F. Madiot. *An Application of Reduced Basis Methods to Core Computation in APOLLO3®*, 2023. HAL preprint cea-04300495.

[3] Y. Conjungo Taumhas, G. Dusson, V. Ehrlacher, T. Lelièvre, F. Madiot. *Reduced basis method for non-symmetric eigenvalue problems : application to the multigroup neutron diffusion equations.* ESAIM : Mathematical Modelling and Numerical Analysis, **58(5)**, 1959, 2023. doi : 10.1051/m2an/2024055.

Contact : amelie.brassart@cea.fr