Ordinary differential equations flows for volume prescribed shape optimization

Amaury BELIERES FRENDO, IRMA - Strasbourg

Charles DAPOGNY, LJK - Grenoble
Victor MICHEL-DANSAC, IRMA - Strasbourg
Yannick PRIVAT, IECL - Nancy

Shape optimization is a central field in applied mathematics and engineering, with applications ranging from aerodynamics to material design. The objective is to find the optimal shape of a domain that minimizes a given criterion, such as energy or resistance, while satisfying geometric constraints like fixed volume, surface area, or minimal height. Classically, solving these optimization problems requires numerical methods that rely on shape derivatives and adjoint calculations, which are computationally expensive and difficult to adapt to complex geometries. To overcome these challenges and the inherent limitations in parallelization found in classical approaches, our objective is to find a good methodology that relies on neural networks and benefits on their specific properties (easy parallelization, automatic differentiation, and a meshless approach). This requires suitable representations for both the solution of the state equation and the shape of the domain in which the equation is defined. For this purpose, we use the DeepRitz method to approximate solutions to the state equation and symplectic neural networks to model the domain shape effectively. We illustrate this approach through an example : minimizing the Dirichlet energy of a domain under a volume constraint [1]. We will then propose an extention of this work for odd dimensions.

[1] A. B. Frendo, E. Franck, V. Michel-Dansac, Y. Privat. *Volume-preserving geometric shape optimization of the dirichlet energy using variational neural networks.* Neural Networks, **184**, 106957, 2025.

Contact : `belieres@unistra.fr`