Stabilization of chemostats with mortality rate and substrate dynamics

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In "chemostat"-type population models that incorporate substrate (nutrient) dynamics, the dependence of the birth (or growth) rate on the substrate concentration introduces nonlinear coupling that creates a challenge for stabilization that is global, namely, for all positive concentrations of the biomass and nutrients. This challenge for global stabilization has been overcome in the literature using relatively simple feedback when natural mortality of the biomass is absent. However, under natural mortality, it takes fortified, more complex feedback, outside of the existing nonlinear control design toolbox, to avoid biomass extinction from nutrient-depleted initial conditions. Such fortified feedback, the associated control Laypunov function design, and Lyapunov analysis of global stability are provided in [1]. We achieve global stabilization for two different chemostat models : (i) a lumped model, with two state variables, and (ii) a three-state model derived from an age-structured infinite-dimensional model. The proposed feedback stabilizers are explicit, applicable to both the lumped and the age-structured models, and coincide with simple feedback laws proposed in the literature when the mortality rate is zero. Global stabilization means subject to constraints : all positive biomass and nutrient concentrations are within the region of attraction of the desired equilibrium, and, additionally, this is achieved with a dilution input that is guaranteed to remain positive. For the lumped case with Haldane kinetics, we show that the reproduction rate dominating the mortality (excluding the reproduction and mortality being in balance) is not only sufficient but also necessary for global stabilization. The obtained results are illustrated with simple examples.

Références

[1] I. Karafyllis, E. Loko, M. Krstic, and A. Chaillet. Global stabilization of chemostats with nonzero mortality and substrate dynamics. *Submitted to IEEE Transaction on Automatic Control*, 2025.

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