Distributionally Robust Optimization applied to Shape Optimization Problem

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Shape optimization aims to determine the optimal design of a mechanical structure based on a given performance criterion. These problems are typically formulated using physical parameters, such as material coefficients or viscosity, which are often difficult to estimate. A major challenge arises because a design optimized for a specific set of parameters may significantly deteriorate under slight variations in these parameters.

To address this issue, various models have been developed, starting with worst-case approaches [1], in which uncertain parameters are assumed to be bounded within a known range. However, these approaches are hindered by their min-max structure, which is theoretically complex to handle. Additionally, robust designs derived from worst-case optimization may be overly conservative, as they are tailored to extreme scenarios that are often unlikely to occur. To alleviate this limitation, stochastic methods have been introduced [4], assuming that uncertain parameters follow a probabilistic distribution. Nevertheless, these models rely on precise knowledge of the probability distribution, which is typically unavailable in practice.

In the context of convex optimization, alternative strategies have been developed to circumvent the lack of distributional knowledge. Distributionally robust optimization (DRO) aims to optimize the mean performance with respect to the worst-case probability distribution within a set of plausible distributions, often inferred from empirical observations. The Wasserstein distance has been widely used to quantify proximity between distributions, enabling computationally efficient and well-posed formulations under reasonable assumptions [2][6]. Additionally, ambiguity sets based on moment uncertainty provide an alternative approach to modeling distributional uncertainty [5].

This talk will explore the application of distributionally robust optimization in the context of shape and topology optimization [3]. Through numerical examples, we will demonstrate that distributionally robust solutions not only achieve strong performance with respect to reference parameters but also effectively account for likely worst-case realizations. Furthermore, we will discuss extensions to alternative methods for handling uncertainty, such as failure probability estimation using the notion of condtional value at risk, prominent concept in finance.

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