A learning-based approach for traffic state reconstruction from limited data

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We propose an efficient method for reconstructing traffic density with low penetration rate of probe vehicles, relying solely on the initial and final positions of a small subset of cars. Due to the scarcity of collected data, we generate artificial trajectories using microscopic dynamical systems [2] and design a machine learning model to approximate the traffic density.
In [1], the authors have incorporated physics-informed neural networks (PINNs) to enforce conservation laws via PDE-derived Lagrangian terms, which necessitated real-time measurements. In contrast, our method requires less information, leveraging only the initial and final positions of probe vehicles, thus simplifying data collection.
Our method uses a residual network (ResNet) to analyze traffic dynamics. The learning process is set up as a constrained optimization problem

$$\begin{aligned} &\underset{\alpha^N}{\text{minimize}} \quad \frac{1}{2}\|x(T)-\bar{y}\|^2 \\ \text{s.t.}\quad &\dot{x}(t) = V\left(W_{\alpha^N}x(t)+b_{\alpha^N}(t)\right), \quad t\in[0,T], \\ &x(0)=\bar{x}, \\ &\alpha^N\in\mathcal{A}_N \end{aligned} \tag{1}$$

where W_{α^N} and b_{α^N} are the weights and biases of the network while nonlinear map V acts as a physics-grounded activation function and $\mathcal{A}_N$ is the set of feasible parameters.
Starting from observed initial positions, the network predicts future states while incorporating physics-based principles of traffic flow.
Ultimately, inspired by [3], we prove that, when using only synthetic data from dynamical systems, our learned traffic density approximation converges to the LWR model as the number of vehicles increases.

Proposition 1. *Under some assumptions, the approximate density*

$$\rho^N(t,x) = \sum_{i=0}^{N-1} \frac{\bar{\alpha}_i^N L}{N(x_{i+1}(t)-x_i(t))}\chi_{[x_i(t),x_{i+1}(t))}(x), \quad x\in\mathbf{R}, \quad t\in[0,T], \tag{2}$$

where $\bar{\alpha}^N$ is a solution to (1) *converges to the unique entropy solution ρ of the LWR model*

$$\begin{aligned} &\frac{\partial\rho}{\partial t}(t,x)+\frac{\partial f(\rho)}{\partial x}(t,x)=0, \quad x\in\mathbf{R}, \quad t\in[0,T], \\ &\rho(0,x)=\bar{\rho}(x), \quad x\in\mathbf{R}. \end{aligned} \tag{3}$$

Références

[1] Barreau, M., Aguiar, M., Liu, J., Johansson, K.H. "Physics-informed Learning for Identification and State Reconstruction of Traffic Density." September 2021.

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[3] Di Francesco M., Rosini M.D. "Rigorous derivation of nonlinear scalar conservation laws from follow-the-leader type models via many particle limit." September 2015.

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