Poster de recherche

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Title : Nonlinear reaction-diffusion problem with membrane conditions

Let consider the domain $\Omega = \Omega_1 \cup \Omega_2 \subset R^2$ with internal interface Γ and boundary $\partial\Omega = \Gamma_1 \cup \Gamma_2$ where $\Gamma_1 = \partial\Omega_1 \backslash \Gamma$ and $\Gamma_2 = \partial\Omega_2 \backslash \Gamma$.

On the two domains $Q_T^1 := (0,T) \times \Omega_1$ and $Q_T^2 := (0,T) \times \Omega_2$, let consider a nonlinear reaction-diffusion membrane problem and I am interested in the effect of the membrane and its permeability realised by the interface conditions describing the crossing flux through Γ as

$$\nabla\varphi(u_1) \cdot n = \nabla\varphi(u_2) \cdot n = \varphi(u_2) - \varphi(u_1)$$

where $u_i = u_i(t,x)$ represents a density in Ω_i and φ is a continuous and strictly increasing function. We observe that with $\varphi(x) = kx$, we obtain the so-called Kedem-Katchalsky conditions, widely used in biophysical contexts, where k is called permeability coefficient.

This work extends Giorgia Ciavolella's study on Kedem-Katchalsky conditions by generalizing the framework to nonlinear diffusion and interface conditions through a function φ.

To illustrate the behavior of the model and establish the existence of weak solutions, I use a finite volume numerical scheme.

References

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