

Numerical Convergence of Lattice Boltzmann Schemes for Parabolic Problems Under Monotonicity Conditions

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Many applications require solving parabolic equations, such as drift-diffusion equations for cold plasmas. Recent studies (see [4], [5]) have introduced Lattice Boltzmann schemes capable of handling parabolic problems, providing some stability results in simple cases with constant coefficients. On the other hand, the works [2] and [1] have demonstrated that Lattice Boltzmann schemes for hyperbolic cases converge when monotonicity conditions on the relaxation are satisfied.

In this talk, we investigate Lattice Boltzmann schemes inspired by the models developped in [3] and study numerical convergence under similar monotonicity conditions. Our objective is to find schemes capable of solving a broader class of parabolic problems and test these schemes in practical applications such as drift diffusion problems from plasma physics.

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