Stability of the trivial equilibrium in degenerate monostable reaction-diffusion equations

S. TRÉTON, Laboratoire de Mathématiques de Bretagne Atlantique - Brest

This talk addresses the long-term behavior of reaction-diffusion equations $\partial_t u = \Delta u + f(u)$ in $\mathbb{R}^N$, where the growth function f behaves as u^{1+p} when u is near the origin.
Specifically, we are interested in the persistence *versus* extinction phenomena in a population dynamics context, where the function u represents a density of individuals distributed in space.
The degenerate behavior $f(u) \sim u^{1+p}$ near the null equilibrium models the so-called Allee effect, which penalizes the growth of the population when the density is low. This effect simulates factors such as inbreeding, mating difficulties, or reduced resistance to extreme climatic events.
We will begin the presentation by discussing a result linking the questions of persistence and extinction with the dimension N and the intensity of the Allee effect p, as established in the classical paper by Aronson and Weinberger (1978).
This result is closely related to the seminal work of Fujita (1966) on blow-up *versus* global existence of solutions to the superlinear equation $\partial_t u = \Delta u + u^{1+p}$.
Following these preliminary results, we will focus on a reaction-diffusion system involving a "heat exchanger", where the unknowns are coupled through the diffusion process, integrating super-linear and non-coupling reactions.
An analysis of the solution frequencies for the purely diffusive heat exchanger will allow us to estimate its "dispersal intensity", which is key information for addressing blow-up *versus* global existence in such semi-linear problems.
This work represents a first step toward Fujita-type results for systems coupled by diffusion and raises several open questions, particularly regarding the exploration of more intricate diffusion mechanisms.

Contact : `samuel.treton@univ-brest.fr`