Control of collision orbits

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In the n-body problem, the forces acting between particles approach infinity when the mutual distances approach zero. Therefore, at collision, the equations of motion have singularities. Since Levi-Civita and Sundman,the double collision has been "regularized", i.e. the singularity has been made to disappear by means of algebraic transformations. Levi-Civita obtained first a regularizing transformation of the Kepler problem based on the map $z \to z^2$ of the complex plane. This conformal map sends the orbits of the harmonic oscillator in the ones of the Kepler problem. The coordinates transformation is coupled with a slowing down of the motion by means of a time change defined by $d\tau = 1/|z|dt$.
Based on classical results, the purpose of the work is the application of the regularization theory to optimal control. We consider the control of a spacecraft under the attraction of one or more bodies, where the control is the thrust, and with the goal of minimizing the integral of some function f. If $f = 1$, the setting is a time-minimization problem. The *Pontryagin's maximum principle (PMP)* allows us to reduce the differential system to the analysis of an Hamiltonian system for finding optimal solutions. However, the system exhibits singularities in zero, and thus the PMP fails in the study of collision orbits. By applying the Levi-Civita regularization to the controlled system, we obtain an affine system (in the new time τ) where the vector fields are complete and C^∞.

[1] A. Albouy. *The levi-civita regularizations*, 2023. New Frontiers of Celestial Mechanics : theory and applications.

[2] A. Albouy, L. Zhao. *Darboux inversions of the kepler problem.* Regular and Chaotic Dynamics, **27(3)**, 253–280, 2022.

[3] V. Arnold, V. Kozlov, A. Neishtadt. *Mathematical Aspects of Classical and Celestial Mechanics.* Springer-Verlag Berlin Heidelberg, 2006.

[4] T. Levi-Civita. *Sulla regolarizzazione del problema piano dei tre corpi.* Rend. Mat. Acc. Lincei (5), **24**, 1915.

[5] T. Levi-Civita. *Sur la régularisation du problèm des trois corps.* Rend. Mat. Acc. Lincei, **24**, 1920.

[6] J. Moser. *Regularization of kepler's problem and the averaging method on a manifold.* Comm. Pure Appl. Math., **23**, 1970.

[7] P. Painlevé. *Leçons sur la théorie analytique des équations differentielles.* Hermann, 1897.

[8] C. Siegel, J. Moser. *Lectures on Celestial Mechanics.* Grundlehren Math. Wiss., 1971.

[9] K. Sundman. *Recherches sur le problème des trois corps*, vol. 34. Acta Societatis Scientiarum Fennicæ, 1907.

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