Rigorous Floating-Point Error Analysis of a Linear Recurrence Computing the Probability of Collision Between Two On-Orbit Objects

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In [1], we design a rigorous floating-point error analysis of an algorithm due to Serra et al. [3] for the computation of the orbital collision probability in the short-term encounter model. This algorithm reduces the numerical computation of the collision probability to that of the sum of a series whose coefficients are generated by a linear recurrence relation. While its numerical stability was experimentally observed and a truncation error bound was derived, the evaluation error caused by the accumulation of rounding errors when unrolling the recurrence in floating-point arithmetic was not investigated. Furthermore, a naive evaluation in interval arithmetic does not provide a satisfactory solution, as it is quickly subject to significant overestimations of the error.

Our work provides an a priori bound on the relative evaluation error, of the form $(N + A)u + o(u)$, where N is the truncation order, u is the round-off unit, A is an explicit constant that depends on the problem parameters (and dominates N in practice), and $o(u)$ stands for small compared to u, explicitly bounded terms. The key ingredient, based on a method developed by Mezzarobba in [2], is the observation that the generating series of the errors affecting each individual term is the solution to a perturbed form of a differential equation satisfied by the Laplace transform of a function related to the collision probability. Hence, the analytical behavior of the error is essentially similar to that of the probability function to be evaluated, up to a factor u, leading to such a relative and reasonably tight error bound.

[1] D. Arzelier, F. Bréhard, M. Joldes, M. Mezzarobba. *Rounding error analysis of an orbital collision probability evaluation algorithm.* In *2024 IEEE 31st Symposium on Computer Arithmetic (ARITH)*, pp. 96–103, 2024. doi :10.1109/ARITH61463.2024.00025.

[2] M. Mezzarobba. *Rounding error analysis of linear recurrences using generating series.* ETNA - Electronic Transactions on Numerical Analysis, **58**, 196–227, 2023. doi :10.1553/etna_vol58s196.

[3] R. Serra, D. Arzelier, M. Joldes, J. Lasserre, A. Rondepierre, B. Salvy. *Fast and accurate computation of orbital collision probability for short-term encounters.* Journal of Guidance Control and Dynamics, **39(9)**, 1009–1021, 2016.

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