A Hybrid Reduced-Order Model for Accelerating Hydrodynamic Simulations

A. MOUSSADDAK, EDF R&D,IMB, Université de Bordeaux
INRIA Bordeaux Sud-Ouest - Saclay, Talence

A. DECOENE, IMB, Université de Bordeaux - Talence
C. GOEURY, Laboratoire d'Hydraulique Saint-Venant (LHSV),
ENPC, Institut Polytechnique de Paris, EDF R&D - Chatou

A. PONÇOT, EDF R&D - Saclay
T. TADDEI, IMB, Université de Bordeaux & INRIA Bordeaux Sud-Ouest - Talence

Water resource management and flood forecasting are crucial societal and financial stakes that require a solid capacity of flow depth estimation. Numerical simulations are essential for obtaining precise flow depth predictions, which facilitate improved planning and response strategies. High-fidelity solvers, such as TELEMAC-2D [4] which is part of the openTELEMAC numerical platform (`www.opentelemac.org`), utilize a numerical approach that couples residual distributive schemes with finite element discretizations and specialized treatments for wetting-drying phenomena. However this approach renders the extraction of operators necessary for conventional projection-based reduced-order models (e.g., Galerkin or Petrov-Galerkin) extremely challenging.
To address these limitations, we propose a hybrid formulation that combines Galerkin projection with a data-fitted closure model. Our approach is inspired by the CD-ROM method [2] and the Mori-Zwanzig formalism [3, 1], which together demonstrate that incorporating the solution's history can recover high-frequency dynamics lost during truncation. In this approach, TELEMAC-2D is exclusively used to generate snapshots, while the reduced-order model is constructed using a *best-knowledge solver* that solves the shallow water equations without the diffusion term via a cell-centered finite volume scheme (in contrast to TELEMAC-2D's node-centered approach). A notable advantage of this strategy is its applicability when the high-fidelity solver is available only as a black box ; provided that the governing equations are known, an alternative solver can be developed and employed within this framework.
This solver strategy introduces multiple error sources : the truncation error from Proper Orthogonal Decomposition (POD) [5], modeling errors arising from neglecting the diffusion term, and discretization errors due to differences in numerical schemes between solvers. To attenuate these discrepancies, we employ a closure model based on Neural Network. In our approach, parameter dependencies are embedded into the encoder networks to better capture the wavefront dynamics. While these modifications are expected to improve the model's performance across various operating conditions, further evaluation is needed to confirm these benefits.
Furthermore, hyper-reduction techniques are employed to drastically reduce the online computational cost without compromising the essential physical phenomena of the flow. The recent analyses presented in [4] underscore the complexity of using the TELEMAC-2D operator for the development of projection-based reduced-order models and thus highlight the value of our approach. Overall, this approach offers a promising pathway for rapid flood modeling and real-time hydraulic decision-making, bridging the gap between computational efficiency and physical fidelity.

[1] Y. T. Lin, Y. Tian, M. Anghel, D. Livescu. *Data-driven learning for the Mori-Zwanzig formalism : a generalization of the Koopman learning framework.* `https://arxiv.org/abs/2101.05873`, 2021. ArXiv preprint arXiv :2101.05873.

[2] E. Menier, et al. *CD-ROM : Complemented Deep – Reduced Order Model*, 2023. Preprint.

[3] H. Mori. *Transport, collective motion, and Brownian motion.* Progress of Theoretical Physics, **33(3)**, 423–455, 1965.

[4] A. Moussaddak, A. Ponçot, T. Taddei, C. Goeury, A. Decoene. *A review of TELEMAC-2D numerical schemes for modelling two-dimensional free surface flows.* In *Proceedings of the XXXth Telemac User Conference.* Chambéry, France, 2024.

[5] L. Sirovich. *Turbulence and the dynamics of coherent structures.* Quarterly of Applied Mathematics, **45(3)**, 561–571, 1987.

Contact : `abdessamad.moussaddak@edf.fr`