Mean sedimentation speed of a random suspension : First order dilute expansion

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We consider a random suspension of particles falling in a Stokes fluid. Due to the hydrodynamic interactions, the exact trajectory of the particles is rather complex. A first physical relevant quantity is then $\overline{V}$, the mean settling speed of the particles.
The goal of this work is to justify the *Batchelor formula* (1). In the dilute regime $\varphi \to 0$ where φ the volume fraction of particles and in dimension 3, Batchelor formally proved in [1] that

$$\overline{V} \simeq \overline{V}^{(1)}(1 - 6.55\varphi) \tag{1}$$

where $\overline{V}^{(1)}$ corresponds to the settling speed of a single isolated particle.
Our main result is a first order dilute expansion of $\overline{V}$

$$\overline{V} = \overline{V}^{(1)} + \mathcal{R}_1$$

with a quantitative control of the error $\mathcal{R}_1$ in terms of so-called multi-points intensities, an intrinsic notion of dilution, introduced in [3].
A rigorous definition of $\overline{V}$ is provided by [2] in the spirit of stochastic homogenization theory. Our proof relies on a cluster expansion approach and a careful renormalization procedure that will both be explained in detail during the talk.
While our first order expansion is not sufficient to justify the (second order) *Batchelor formula*, we believe that our strategy is robust and can be extended to higher orders.

[1] G. K. Batchelor. *Sedimentation in a dilute dispersion of spheres.* Journal of Fluid Mechanics, **52(2)**, 245–268, 1972. doi :10.1017/S0022112072001399.

[2] M. Duerinckx, A. Gloria. *Sedimentation of random suspensions and the effect of hyperuniformity.* Ann. PDE, **8(1)**, 66, 2022. doi :10.1007/s40818-021-00115-0. Id/No 2.

[3] M. Duerinckx, A. Gloria. *On Einstein's effective viscosity formula*, vol. 7 of *Mem. Eur. Math. Soc.* Berlin : European Mathematical Society (EMS), 2023. doi :10.4171/MEMS/7.

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